On thermal nucleation of quark matter in compact stars

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Abstract. The possibility of a hadron-quark phase transition in extreme astrophysical phenomena such as the collapse of a supernova is not discarded by the modern knowledge of the high-energy nuclear and quark-matter equations of state. Both the density and the temperature attainable in such extreme processes are possibly high enough to trigger a chiral phase transition. However, the time scales involved are an important issue. Even if the physical conditions for the phase transition are favorable (for a system in equilibrium), there may not be enough time for the dynamical process of phase conversion to be completed. We analyze the relevant time scales for the phase conversion via thermal nucleation of bubbles of quark matter and compare them to the typical astrophysical time scale, in order to verify the feasibility of the scenario of hadron-quark phase conversion during, for example, the core-collapse of a supernova.

1. Introduction

Extreme densities and temperatures can be achieved during the highly energetic core-collapse explosion of a type Ib supernova. In fact, temperatures of tens of MeV ($1\text{MeV} \approx 10^{10}\text{K}$) and densities as high as $2n_0$ (n_0 is the nuclear saturation density) can be attained right after the bounce, when the proto neutron star is still hot and lepton-rich.

In general, SN simulations show that the highest central density achieved in the early post-bounce phase is higher than the nuclear saturation density n_0, but hardly higher than $2n_0$. On the other hand, the Equation of State (EoS) for strongly interacting matter at low temperature and high baryon density possibly presents a first order hadron-quark phase transition at the same density scale. Moreover, it has been recently shown [1] that if there is a hadron-quark phase transition in the early post-bounce phase of a core-collapse supernova, it may be decisive for the supernova explosion. A natural question that arises is whether the QCD phase transition may happen or not during an actual supernova. The most important condition is, at least in principle, achievable: the critical density for the transition may be lower than the densities found during the early post-bounce phase ($\leq 2n_0$). However, this condition is not enough for the phase transition to happen, once it takes a finite time to be initiated and completed, and such time must be shorter than a typical time scale of the SN evolution (which we assume to be $\tau_B \lesssim 100$ ms) [2]. This is the central subject of this work.

By calculating the formation rate of quark matter bubbles at given physical conditions, we estimate (or, rather, underestimate) the time scale associated with the phase transition and compare it to the duration of the pre-deleptonization era in a core-collapse supernova τ_B. This comparison can serve as a criterium of feasibility for the formation of quark matter in this scenario, because the process of nucleation must occur inside the time window determined by τ_B.

2. Phenomenological framework

2.1. Equations of state

We discuss the formation of two possible quark phases in the phase transition. In the first case, the phase conversion involves two-flavor quark matter ‡. In the second scenario, we consider a fast production of strange quarks: since we assume critical densities for the phase transition (n_c) of the order of two times the saturation density and temperatures of a few tens of MeV, it is possible that a small seed of strange matter is already present in the system as hyperons and kaons appear [4, 5]. Such particles do not contribute significantly to the pressure or to the energy density in the hadronic phase, but their presence may trigger a phase transition directly to strange quark matter.

To encompass both possibilities for the quark matter phase, we consider two types

‡ Strange quarks will be produced only later, via weak interaction, as suggested in [3].

of high-density EoS, both including the pressure from electrons and neutrinos, which are still present at the early post-bounce phase. In the first case, we consider only *up* and *down* quarks, while in the second we also include a massive *strange* quark. The free parameters of the quark model are the bag constant B, the mass of the strange quark m_s (when present), and the coefficient c [6], that effectively accounts for perturbative QCD corrections to the free gas pressure [7], as follows (terms $\mathcal{O}(m_s^4/\mu_s^4)$ were neglected):

$$p(\{\mu\}) = (1-c)\left[\sum_{i=u,d}\frac{\mu_i^4}{4\pi^2}\right] + (1-c)\frac{\mu_s^4}{4\pi^2} - \frac{3}{4\pi^2}m_s^2\mu_s^2 + \frac{\mu_e^4}{12\pi^2} + \frac{\mu_\nu^4}{24\pi^2} - B \qquad (1)$$

For nuclear matter, we adopt the relativistic mean field model equation of state with the TM1 parametrization [8], often used in supernovae simulations.

The equations of state for nuclear matter and quark matter are calculated under conditions of local charge neutrality, local lepton fraction conservation (i.e., the two phases have the same Y_L), and weak equilibrium. Under these assumptions, the conditions of phase equilibrium are the equality of the total pressure of the two phases $P^H = P^Q$ and condition of chemical equilibrium $\mu_n + Y_L\mu_\nu^H = \mu_u + 2\mu_d + Y_L\mu_\nu^Q$, where μ_n and μ_ν^H are the chemical potentials of neutron and neutrinos within the nuclear phase, and μ_u, μ_d and μ_ν^Q are the chemical potentials of up and down quarks and of neutrinos within the quark phase, respectively [9]. § Finally, the conditions of local charge neutrality and local conservation of the lepton fraction allow us to compute all chemical potentials in terms of one independent chemical potential (see, e.g., [9]).

2.2. Thermal homogeneous nucleation

In first-order phase transitions, if a homogeneous system is brought into instability close enough to the coexistence line of the phase diagram, its dynamics will be dominated by large-amplitude, small-ranged fluctuations, that is, by bubbles of the metastable phase, which eventually grow and complete the phase conversion [10].

The standard theory of thermal nucleation in one-component metastable systems was developed by Langer in the late sixties [11] (see also [12]). In this formalism, a key quantity for the calculation of the rate of nucleation is the coarse-grained free energy functional, that may be cast, in the vicinity of the phase transition, as

$$\Delta F(R) = 4\pi R^2\sigma - \frac{4\pi}{3}R^3(\Delta p), \qquad (2)$$

where σ is the surface tension of the hadron-quark interface. From Eq. (2), we can see that $\Delta F(R)$ has a maximum at $R_c \equiv 2\sigma/\Delta p$, the critical radius. The bubbles of this size are called *critical bubbles* and are the smallest bubbles that, once formed, can start to drive the phase conversion. Therefore, to give a quantitative meaning to the process

§ Here we use the zero-temperature equations of state, since a temperature of the order of a few tens of MeV does not alter considerably the equation of state. For a free massless gas, the corrections would be $\mathcal{O}(T^2/\mu^2) \sim 1\%$.

of nucleation, one can calculate the rate Γ of critical bubbles created by fluctuations per unit volume, per unit time:

$$\Gamma = \frac{\mathcal{P}_0}{2\pi} \exp\left[-\frac{\Delta F(R_c)}{T}\right] = T^4 \exp\left[-\frac{16\pi}{3}\frac{\sigma^3}{(\Delta p)^2 T}\right], \qquad (3)$$

where we used (2) and $R_c = 2\sigma/\Delta p$. We make the simple choice $\mathcal{P}_0/2\pi = T^4$, which is an overestimate of the actual pre-factor ‖. Notice also that the influence of the equation of state is present through Δp. Finally, there is a remarkably strong dependence of Γ on the surface tension σ, which will be determinant for the nucleation time scale.

It is convenient to introduce the *nucleation time* τ, defined as the time it takes for the nucleation of one single critical bubble inside a volume of $1 km^3$ in the proto-neutron star core: $\tau \equiv \left(\frac{1}{1km^3}\right)\frac{1}{\Gamma}$. This is the time scale we compare with the duration of the early post-bounce phase of a supernova event, few hundreds of milliseconds, during which the formation of quark matter could trigger the supernova explosion.

3. Results and discussions

3.1. Nucleation times for nonstrange matter

As our first case, we consider the transition from beta-stable nuclear matter to beta-stable quark matter composed of u and d quarks, plus electrons and electron neutrinos, with a fixed lepton fraction $Y_L = 0.4$ and critical baryon density $n_c = 1.5n_0$.

Figure 1 shows the behaviour of the nucleation time of a single critical bubble (as defined in the previous section) versus the density, in units of n_0. As expected, the nucleation time τ has an extremely strong dependence on both the density (notice the logarithmic scale for τ) and on the surface tension. As discussed in the introduction, we consider that nucleation is effective if $\tau < \tau_B \lesssim 100$ ms for $n < 2n_0$. For low values of σ, nucleation becomes feasible at relatively low densities, although the required densities increase steadily as the surface tension rises.

Next, we compare the previously analyzed $c = 0$ case with $c = 0.2$, in the case of $n_c = 1.5n_0$ and $T = 20$ MeV, as displayed in Figure 2, where we also show the influence of the lepton fraction Y_L. The contour lines of Figs. 2-4 (on which the nucleation time is $\tau = 100$ ms) may be considered as "nucleation thresholds": given a value of surface tension, when the rising density crosses one contour line, the nucleation time scale is fast enough for the completion of the phase conversion. Notice that the introduction of interactions drastically increases the nucleation time, so that only for a low value of the surface tension nucleation can be efficient as the density reaches a value close to $2n_0$ ¶. Furthermore, notice that given a fixed density, a decrease in Y_L makes nucleation more efficient, because deleptonization renders nuclear matter less stable.

‖ For a calculation of $\mathcal{P}_0$ see, e.g., [12].
¶ As becomes clear from this analysis, a reliable estimate of σ for cold dense matter is called for.
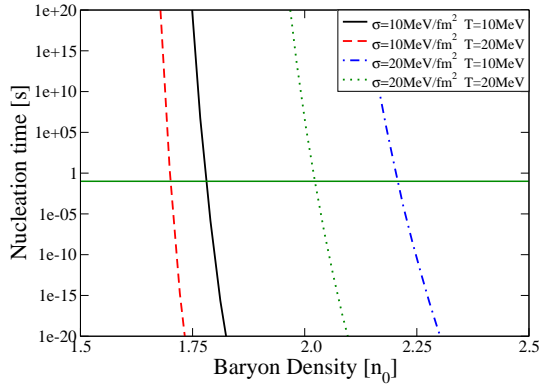
Figure 1. (From [2].) Nucleation time as a function of baryon density for u-d quark matter ($n_c = 1.5n_0$). The horizontal line corresponds to the nucleation threshold $\tau = 100$ms.
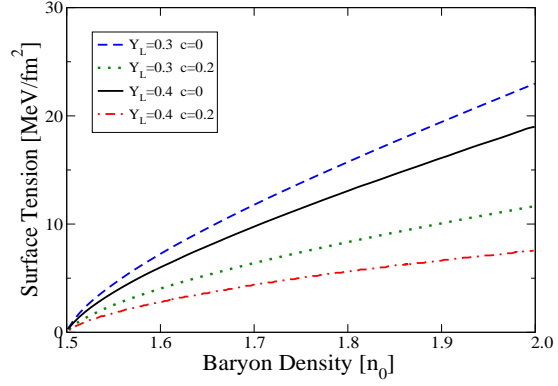

Figure 2. (From [2].) Lines of constant nucleation time ($\tau = 100$ms) as a function of n and σ with noninteracting ($c = 0$) and interacting ($c = 0.2$) u-d quarks, for $Y_L = 0.3$ and $Y_L = 0.4$.

3.2. Nucleation of strange matter

The introduction of strange quarks makes the EoS stiffer, i.e., for a given baryon chemical potential μ the corresponding pressure becomes higher. Once the nuclear EoS is the same, Δp will be higher for a given value of μ, and the nucleation rate will also be higher. Figure 3 shows a comparison between the transition from nuclear matter to either u-d or u-d-s quark matter for two values of the lepton fraction Y_L ($n_c = 1.5n_0$, $T = 20$ MeV, $c = 0$ and $m_s = 0$). As previously discussed, we can notice that in the present case a decrease in Y_L also increases the efficiency of thermal nucleation.

We can also analyze the effect of the strange quark mass on the nucleation time. This can be seen in Figure 4, which also shows how the combined effect of the strange quark mass and of (perturbative) strong interactions can strongly increase the nucleation time ($n_c = 1.5n_0$, $T = 20$ MeV and $Y_L = 0.4$). Notice that the presence of interactions among quarks suggest that the value of σ should not exceed ~ 20 MeV/fm^2 (with the realistic value $m_s = 100$ MeV), if we require nucleation to be efficient.
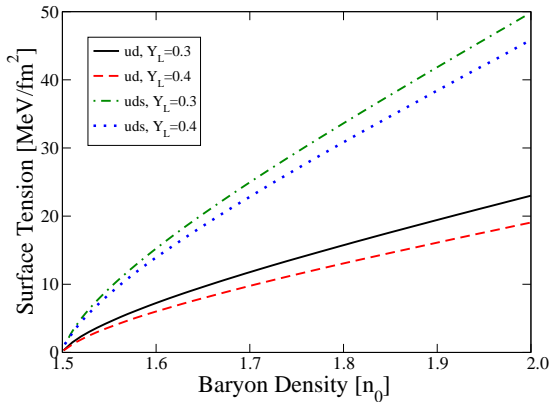

Figure 3. (From [2].) Lines of $\tau = 100$ ms for the transition to u-d or u-d-s quark matter, with lepton fraction $Y_L = 0.3, 0.4$.
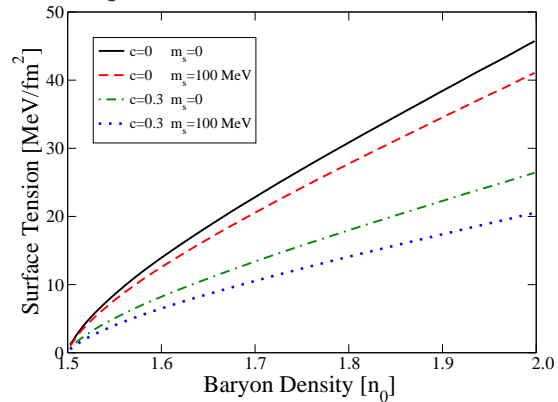

Figure 4. (From [2].) Lines of $\tau = 100$ ms for $c = 0$ and $c = 0.3$, and for quark mass $m_s = 0$ and $m_s = 100$ MeV.

4. Conclusions

We investigated the possibility of formation of quark matter in supernova matter, i.e., for temperatures of the order of a few tens of MeV and in the presence of trapped neutrinos, assuming that the corresponding critical density does not exceed $2n_0$. By calculating the nucleation rate for different values of the free parameters, we argued that thermal nucleation of droplets of the quark phase is possibly the dominant mechanism for the formation of the new phase.

As expected from Eq. (2), the surface tension is the physical quantity which mainly controls the nucleation process and, within our choices of physical conditions (for the EoS we tested), the value of σ should not exceed ~ 20 MeV/fm^2 if the hadron-quark phase transition in core-collapse supernovae occurs via thermal nucleation.

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